

Quantum torsion with non-zero standard deviation: non-perturbative approach for cosmology

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Cosmology with non-perturbative quantum corrections resulting from torsion is considered. It is shown that the evolution of closed, open and flat Universes is changed because of the presence of a non-zero dispersion of quantum torsion. The evolution of a Universe with quantum torsion and with one type of average curvature can be similar to the evolution of a Universe without quantum torsion and with another type of average curvature. For the description of the non-perturbative quantum torsion, a vector field approximation is applied.

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I. INTRODUCTION

The inclusion of torsion is a natural way for the generalization of the connection compatible with the metric. It leads to an affine connection constructed from Christoffel symbols plus a contortion tensor. Einstein-Cartan gravity (ECg) naturally extends general relativity to include the torsion.

ECg has some distinctive features as compared to general relativity. One can show that spin and torsion can avert cosmological singularities for certain spin configurations and for all configurations of matter with spin [1]. In Ref. [2] it was shown that those models violate an energy condition of a singularity theorem. In Ref. [3] the account of quantum torsion to FRW cosmology was considered.

Modern experimental tests of the existence of torsion have led to negative results. In [4, 5] tight bounds on many components of the torsion tensor (mixed-symmetry, trace and axial components) were reported, based on high-precision data from masers and torsion pendulums. While these experiments gave negative results for the search of non-zero torsion, they had nothing to say for the case that torsion is a quantum field with zero expectation value but non-zero dispersion values.

In this letter we discuss the approximation when in the theory of gravity only the torsion is quantized, while the metric remains a classical object. After introducing some basic concepts of non-perturbative quantization (following Heisenberg), we use some approximation (vector field approximation) to obtain the Einstein equations with zero torsion and non-zero torsion dispersion. The vector field approximation means that we approximate the two point Green's function $\langle \hat{Q}_{\mu\nu\rho}(x)\hat{Q}_{\alpha\beta\gamma}(y) \rangle$ by a product of a complex vector field A_μ in the spacetime points x and y (here \hat{Q} denotes the quantum torsion operator).

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II. THE BASICS OF HEISENBERG'S NON-PERTURBATIVE QUANTIZATION: THE APPLICATION TO GRAVITY

According to Heisenberg [6] the quantum operators of the metric $\hat{g}_{\mu\nu}$ and the affine connection $\hat{\Gamma}^\rho_{\mu\nu}$ obey the operator Einstein equations in the Palatini formalism

$$\hat{\Gamma}^\rho_{\mu\nu} = \hat{G}^\rho_{\mu\nu} + \hat{K}^\rho_{\mu\nu}, \quad (1)$$

$$\hat{R}_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}\hat{R} = \varkappa\hat{T}_{\mu\nu}, \quad (2)$$

$$\hat{G}^\rho_{\mu\nu} = \frac{1}{2}\hat{g}^{\rho\sigma}\left(\frac{\partial\hat{g}_{\mu\sigma}}{\partial x^\nu} + \frac{\partial\hat{g}_{\nu\sigma}}{\partial x^\mu} - \frac{\partial\hat{g}_{\mu\nu}}{\partial x^\sigma}\right). \quad (3)$$

Here $\varkappa = 8\pi G/c^4$; $\hat{R}_{\mu\nu}$ is the operator of the Ricci tensor; $\hat{G}^\rho_{\mu\nu}$ are the operators of the Christoffel symbols; $\hat{K}^\rho_{\mu\nu}$ is the operator of the contortion tensor, $\hat{T}_{\mu\nu}$ is the operator of the matter fields and \hat{R} is the operator of the scalar curvature defined in usual manner

$$\hat{R}_{\mu\nu} = \hat{R}^\rho_{\mu\rho\nu}, \quad (4)$$

$$\hat{R}^\rho_{\sigma\mu\nu} = \frac{\partial\hat{\Gamma}^\rho_{\sigma\nu}}{\partial x^\mu} - \frac{\partial\hat{\Gamma}^\rho_{\sigma\mu}}{\partial x^\nu} + \hat{\Gamma}^\rho_{\tau\mu}\hat{\Gamma}^\tau_{\sigma\nu} - \hat{\Gamma}^\rho_{\tau\nu}\hat{\Gamma}^\tau_{\sigma\mu}. \quad (5)$$

The operator of the contortion tensor $\hat{K}^\rho_{\mu\nu}$ is defined via the operator of the torsion tensor $\hat{Q}_{\mu\nu}{}^\rho$

$$\hat{K}^\rho_{\mu\nu} = \hat{Q}^\rho_{\mu\nu} + \hat{Q}_{\mu\nu}{}^\rho - \hat{Q}_\nu{}^\rho{}_\mu. \quad (6)$$

The operator of the torsion tensor is the skew-symmetric part of the affine connection

$$\hat{Q}_{\mu\nu}{}^\rho = \frac{1}{2}\left(\hat{\Gamma}_{\mu\nu}{}^\rho - \hat{\Gamma}_{\nu\mu}{}^\rho\right). \quad (7)$$

The non-perturbative quantization for Einstein gravity means that the quantum operators $\hat{\Gamma}, \hat{g}$ obey the operator Einstein equations (1), (2).

III. VECTOR FIELD APPROXIMATION FOR NON-PERTURBATIVE QUANTIZATION OF TORSION

In the approximation presented here we reserve the metric as a classical object and we quantize the torsion only. For simplicity we will consider the skew-symmetric torsion

$$\hat{K}_{\rho\mu\nu} = \hat{Q}_{\rho\mu\nu} = \hat{Q}_{[\rho\mu\nu]}. \quad (8)$$

In our approach we consider the torsion with zero expectation value

$$\langle \hat{Q}^\rho_{\mu\nu} \rangle = 0 \quad (9)$$

but with non-zero dispersion

$$\langle (\hat{Q}^\rho_{\mu\nu})^2 \rangle \neq 0. \quad (10)$$

We use a vector field approximation

$$\langle \hat{Q}_{\rho_1\mu_1\nu_1}(x_1)\hat{Q}_{\rho_2\mu_2\nu_2}(x_2) \rangle = \epsilon_{\rho_1\mu_1\nu_1\alpha}\epsilon_{\rho_2\mu_2\nu_2\beta}A^\alpha(x_1)A^\beta(x_2). \quad (11)$$

The expectation values of the Ricci and scalar curvature operators with the vector field approximation for the non-perturbative gravity quantization will be

$$\langle \hat{R}_{\mu\nu} \rangle = \tilde{R}_{\mu\nu} - \langle \hat{Q}^\rho_{\mu\sigma}\hat{Q}^\sigma_{\rho\nu} \rangle = \tilde{R}_{\mu\nu} - \epsilon^\rho_{\mu\sigma\alpha}\epsilon^\sigma_{\rho\nu\beta}A^\alpha A^\beta = \tilde{R}_{\mu\nu} + 2(g_{\mu\nu}A_\alpha A^\alpha - A_\mu A_\nu), \quad (12)$$

$$\langle \hat{R} \rangle = \tilde{R} + 6A^\mu A_\mu, \quad (13)$$

where $\tilde{R}_{\mu\nu}$ and \tilde{R} are the Ricci tensor and scalar curvature obtained from the metric in the standard way. Then the Einstein equations are

$$\tilde{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\tilde{R} - (g_{\mu\nu}A_\alpha A^\alpha + 2A_\mu A_\nu) = \varkappa T_{\mu\nu}, \quad (14)$$

where for simplicity we consider classical fields on the RHS. Now we would like to obtain an equation for the vector field A_μ , approximately describing non-perturbative quantum gravitational effects. In order that the Einstein equations

$$\left\langle \hat{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\hat{R} \right\rangle = \varkappa T_{\mu\nu} \quad (15)$$

are not overdetermined we demand that

$$\left\langle \left(\hat{R}_\nu^\mu - \frac{1}{2}\delta_\nu^\mu \hat{R} \right) \right\rangle_{;\mu} = 0 \quad (16)$$

since

$$T_{\nu;\mu}^\mu = 0. \quad (17)$$

Taking into account that

$$\left(\tilde{R}_\nu^\mu - \frac{1}{2}\delta_\nu^\mu \tilde{R} \right)_{;\mu} = 0 \quad (18)$$

we obtain the desired equation for the vector field A_μ

$$(\delta_\nu^\mu A^\alpha A_\alpha + 2A^\mu A_\nu)_{;\mu} = 0 \quad (19)$$

or

$$(A^\alpha A_\alpha)_{;\nu} + 2A_{;\mu}^\mu A_\nu + 2A^\mu A_{\nu;\mu} = 0. \quad (20)$$

IV. NON-PERTURBATIVE QUANTUM TORSION IN COSMOLOGY

Now we would like to consider cosmology with quantum corrections coming from the torsion. We take the metric for closed, open and flat Universes in the form

$$ds^2 = \begin{cases} a^2(\eta) \{ d\eta^2 - [d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\varphi^2)] \}, & \text{closed Universe} \\ a^2(\eta) \{ d\eta^2 - [d\chi^2 + \sinh^2 \chi (d\theta^2 + \sin^2 \theta d\varphi^2)] \}, & \text{open Universe} \\ a^2(\eta) \{ d\eta^2 - [dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)] \}, & \text{flat Universe.} \end{cases} \quad (21)$$

We take the vector field A_μ as follows

$$A_\mu = (\phi(\eta), 0, 0, 0). \quad (22)$$

The averaged Einstein equations (14) in the presence of matter in the form of dust are

$$\frac{a'^2}{a^2} + (k - \phi_0^2) = \varkappa \frac{\varepsilon_0}{3a}, \quad (23)$$

$$2\frac{a''}{a} + \frac{a'^2}{a^2} + (k - \phi_0^2) = 0, \quad (24)$$

where the prime denotes differentiation with respect to η ; ε_0 is a constant and $k = +1, -1, 0$ for closed, open and flat Universes. In Eqs. (23), (24) it is taken into account that Eq. (20) with the ansatz (22) yields

$$\phi' = 0, \quad (25)$$

and the solution of this equation is

$$\phi = \phi_0 = \text{const.} \quad (26)$$

We see that the quantum torsion correction ϕ_0^2 changes the evolution of the Universe by adding the term $-\phi_0^2$ to the Friedman equations. In hydrodynamical language, this term corresponds to a perfect fluid in the form of the so-called “string gas” or “gas of strings” having an equation of state $p = -\varepsilon/3$. Since in form the term with ϕ_0^2 is similar to that with k (which describes the spatial curvature for the metric (21)), it is natural to introduce some geometrical interpretation for this term [7]. Below we consider the consequences of the presence of this term on the evolution of three models corresponding to closed, open and flat Universes.

A. Closed Universe

The most interesting case is a closed Universe, $k = 1$. Let us here consider three cases: the first case with $\phi_0^2 = 1$; the second one with $\phi_0^2 > 1$ and the third one with $\phi_0^2 < 1$:

- $\phi_0^2 = 1$. In this case Eq. (23) tells us that the evolution of the scale factor $a(t)$ is the same as for a flat (non-torsion) Universe. The parametric dependence $a(t)$ is as follows

$$a(\eta) = \frac{\varkappa\varepsilon_0}{12}\eta^2, \quad (27)$$

$$ct(\eta) = \frac{\varkappa\varepsilon_0}{36}\eta^3. \quad (28)$$

The scale factor evolves as

$$a = \left(\frac{3\varkappa\varepsilon_0}{4} \right)^{1/3} (ct)^{2/3}. \quad (29)$$

Thus the evolution of a closed quantum-torsion Universe will not differ from the evolution of a flat (non-torsion) Universe.

- $\phi_0^2 > 1$. In this case the term $1 - \phi_0^2$ in Eq. (23) is negative and the closed Universe evolves as an open (non-torsion) Universe. The parametric dependence $a(t)$ is as follows

$$a(x) = \frac{\varkappa\varepsilon_0}{6(\phi_0^2 - 1)} (\cosh x - 1), \quad (30)$$

$$t(x) = \frac{\varkappa\varepsilon_0}{6(\phi_0^2 - 1)c} (\sinh x - x), \quad (31)$$

where $x = \eta\sqrt{\phi_0^2 - 1}$. Considering large values of $x \gg 1$ yields

$$a \approx ct. \quad (32)$$

This expression coincides with the expression for an open (non-torsion) Universe. In turn, close to the Big Bang, i.e., when $x \ll 1$,

$$a \approx 3 \left[\frac{\varkappa\varepsilon_0}{36(\phi_0^2 - 1)} \right]^{1/3} (ct)^{2/3}, \quad (33)$$

i.e., one has the expression similar to that of the case of open Universe without the quantum torsion.

- $\phi_0^2 < 1$. In this case the term $1 - \phi_0^2$ in equation (23) is positive and the closed Universe evolves similar to the closed (non-torsion) Universe. Here the parametric dependence $a(t)$ is the following

$$a(x) = \frac{\varkappa\varepsilon_0}{6(1 - \phi_0^2)} (1 - \cos x), \quad (34)$$

$$t(x) = \frac{\varkappa\varepsilon_0}{6(1 - \phi_0^2)c} (x - \sin x), \quad (35)$$

where $x = \eta\sqrt{1 - \phi_0^2}$. Again, close to the Big Bang $a(t)$ is

$$a \approx 3 \left[\frac{\varkappa\varepsilon_0}{36(1 - \phi_0^2)} \right]^{1/3} (ct)^{2/3}. \quad (36)$$

B. Open Universe

In this case the parameter $k = -1$ and the quantum torsion correction $-\phi_0^2$ are both negative. In this case the parametric dependence $a(t)$ is the following

$$a(x) = \frac{\varkappa\varepsilon_0}{6(\phi_0^2 + 1)} (\cosh x - 1), \quad (37)$$

$$t(x) = \frac{\varkappa\varepsilon_0}{6(\phi_0^2 + 1)c} (\sinh x - x), \quad (38)$$

where $x = \eta\sqrt{\phi_0^2 + 1}$. For large values of $x \gg 1$

$$a \approx ct. \quad (39)$$

For $x \ll 1$ we have

$$a \approx 3 \left[\frac{\varkappa \varepsilon_0}{36(\phi_0^2 + 1)} \right]^{1/3} (ct)^{2/3}. \quad (40)$$

C. Flat Universe

In this case the parameter $k = 0$. The parametric dependence $a(t)$ is the following

$$a(x) = \frac{\varkappa \varepsilon_0}{6\phi_0^2} [\cosh x - 1], \quad (41)$$

$$t(x) = \frac{\varkappa \varepsilon_0}{6\phi_0^2 c} [\sinh x - x], \quad (42)$$

where $x = \eta\phi_0$. For large values of $x \gg 1$

$$a \approx ct. \quad (43)$$

Close to the Big Bang (where $x \ll 1$) $a(t)$ is

$$a \approx 3 \left[\frac{\varkappa \varepsilon_0}{36\phi_0^2} \right]^{1/3} (ct)^{2/3}. \quad (44)$$

V. CONCLUSIONS

In this paper we have investigated the role of quantum torsion on the evolution of the Universe. We have considered a form of torsion with zero expectation value but with non-zero dispersion. We have used the approximation where the expectation value of the product of the torsion operator in two points is *approximated* by a vector field. As a result, we have shown that the quantum torsion may lead to a qualitative change of the evolution of the Universe. For example, a closed Universe with fluctuating quantum torsion may have an evolution similar to a closed, open or flat (non-torsion) Universe, depending on the value of the quantum fluctuation dispersion of the torsion. One can note that using a “scalar approximation” for the quantum torsion leads to the dark energy [8].

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